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Radiation Characteristics of Traveling-Wave Antennas Excited by Nonsinusoidal Currents

NIE ZAIPING

Abstract—**In this paper, the idea about traveling-wave antennas excited by nonsinusoidal currents for high-resolution exploration is advanced. First the analytic expressions of the radiation waveforms in the far zone produced by a single transient of current traveling along a straight wire are derived in time domain. Then the radiation characteristics of traveling-wave antennas excited by nonsinusoidal current, such as the mean-power pattern functions and the directivity, are given, and the space waveform control and radiation efficiency are discussed. Based on previous results, the radiation waveforms of dipole antennas excited by pulse current are obtained from time-domain analysis. In a practical example, comparing these waveforms with corresponding waveforms obtained from frequency-domain analysis and measured data in [4] and [13], good agreements are obtained in various radiation directions. Some errors in [14] are also pointed out and corrected.**

Key Words—**Traveling-wave antennas, radiation characteristics, nonsinusoidal current excitation, analysis, time domain.**

I. INTRODUCTION

THE PURPOSE of developing radar technique is always to obtain high-resolution high-precision target discrimination and a good image. This requires an increase of the bandwidth of the radiated signal. The ultimate form is to use directly a video-pulse signal. Because of this, research on nonsinusoidal-wave radar techniques is increasing. Specifically, from the 1970s, Harmuth [1]-[3] has made important nonsinusoidal waves. Schmitt *et al.* [4], Abo-Zena and Beam [5], Fralick [6], and Chufang [7] have also done much successful work in the research on nonsinusoidal-wave radiation theory. The Rome Air Development Center [8], Stanford Research Institute [6], and Ohio State University [9] have conducted impressive experiments on nonsinusoidal-wave radiation and scattering. As a practical application of nonsinusoidal waves, Cook [10], [11] and Moffatt *et al.* [12] devoted themselves to the design of video-pulse into-the-ground radar. At present, at least 40 pulse-into-the-ground radars are working in 10 countries.

The most interesting and the most difficult problem in nonsinusoidal-wave radar is how to transmit and receive pulses effectively and directively with high fidelity. Literatures published so far have paid great attention to the broad-band requirements of the input characteristics and the radiation characteristics of pulse antennas, but often neglected another important requirement, i.e., there should be no reflection of the exciting current between source, feeder, and antenna, as well as at different parts of the antenna. Actually, in radar applications, we are usually interested only in radiating in space short pulses with long periodic repetition time and do not require the pulse to have the same time variation as the

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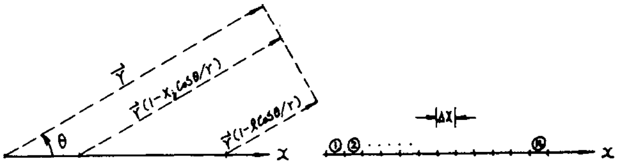


(a) (b)

Fig. 1. Geometry of single traveling-wave antenna.

exciting current in the antenna. Furthermore, if we are able to know and control the distortion of the radiated pulse, then this distortion can be used for waveform transformation and space pulse compression. This aspect will be discussed later. However, for high-resolution radar systems, the damped oscillation caused by the transmission back and forth of the pulse current in the antenna must be prevented. Otherwise, the radar resolution will worsen greatly, false images will occur, and the discrimination of the target will become difficult.

Hence, in this paper, the idea about the traveling-wave antenna excited by a nonsinusoidal signal for high-resolution exploration is advanced. First, the analytic expressions of the radiation waveform in the far zone produced by a single transient of current traveling along a straight wire are derived in the time domain, then the mean-power pattern functions of the traveling-wave antenna are given, and the radiation waveform control and radiation efficiency are discussed. Based on previous results, the radiation waveforms of a dipole antenna excited by pulse currents are obtained from time-domain analysis. In a practical example, comparing these waveforms with corresponding waveforms obtained from frequency-domain analysis and measured data in [4] and [13], good agreements are obtained in various radiation directions. These agreements demonstrate previous results for the traveling-wave antenna are correct. Finally, some errors in [14] are also pointed out and corrected.

II. RADIATION CHARACTERISTICS OF A SINGLE TRANSIENT OF CURRENT TRAVELING ALONG A WIRE

Consider the traveling-wave linear antenna with length l, as shown in Fig. 1(a). This antenna can be treated as the one-dimensional traveling-wave array of many current elements (Hertzian dipoles) with length Δx arranged coaxially and fed in series, as shown in Fig. 1(b). Assume point $x = 0$ is the feed point and $i(0, t)$ is the exciting current at $x = 0$; then the exciting current of Hertzian dipole Δx_j at $x = x_j$ is expressed as follows:

$$i(x_j, t) = i\left(0, t - \frac{x_j}{c_0}\right) \tag{1}$$

where c_0 is the propagation velocity of the current along direction x. If account is taken of the time delay caused by wave propagation in space, the electric field strength produced by this Hertzian dipole at an observation point $p(r, \Theta, \varphi)$ is [1]

$$E\left(\frac{1}{r}, \Theta, t\right) = E(\Theta, t)\frac{1}{r}\frac{r \times (r \times s)}{sr^2} \tag{2}$$

where

$$E(\Theta, t) = \frac{Z_0}{4\pi c}\frac{d}{dt}\, i\left(0, t - \frac{x_j}{c_0} - \frac{r - x_j\cos\Theta}{c}\right)\cdot \Delta x_j$$

c is the wave velocity in the surrounding space and Z_0 is the wave impedance. In a spherical coordinate system, (2) can be written as

$$E\left(\frac{1}{r}, \Theta, t\right) = E\left(\frac{1}{r}, \Theta, t\right)\hat{a}_\Theta \tag{3}$$

where

$$E\left(\frac{1}{r}, \Theta, t\right) = \frac{Z_0 \sin\Theta}{4\pi cr}\frac{d}{dt} i\left(0, t - \frac{x}{c_0} - \frac{r - x_j\cos\Theta}{c}\right)\cdot \Delta x_j$$

a_Θ is the unit vector in the increasing Θ direction.

Thus the electric field strength in the far zone produced by the traveling-wave array with n elements is

$$E_l\left(\frac{1}{r}, \Theta, t\right) = E_l\left(\frac{1}{r}, \Theta, \Theta\right)\hat{a}_\Theta \tag{4}$$

where

$$E_l\left(\frac{1}{r}, \Theta, t\right) = \frac{Z_0 \sin\Theta}{4\pi cr}\sum_{j=1}^{n}\frac{d}{dt} \cdot i\left(0, t - \frac{x_j}{c_0} - \frac{r - x_j\cos\Theta}{c}\right)\Delta x_j.$$

Let $\Delta x_j \to 0$, $n \to \infty$; the field in the far zone produced by a single transient of current traveling along a linear antenna is

$$E_l\left(\frac{1}{r}, \Theta, t\right) = \frac{Z_0 \sin\Theta}{4\pi cr}\int_0^l \frac{d}{dt} \cdot i\left(0, t - \frac{x}{c_0} - \frac{r - x\cos\Theta}{c}\right)dx. \tag{5}$$

Assume the exciting current $i(0, t)$ at the feed point is a linear transition with transient time ΔT, as shown in Fig. 2(a). For this single current transition

$$\frac{di(0, t)}{dt} = \begin{cases} 0, & t < 0 \\ I/\Delta T, & 0 \leqslant t \leqslant \Delta T \\ 0, & t > \Delta T. \end{cases} \tag{6}$$

Therefore

$$\frac{d}{dt} i\left(0, t-\frac{x}{c_0}-\frac{r-x\cos\Theta}{c}\right) = \begin{cases} 0 & x<\dfrac{ct-r-c\Delta T}{\xi-\cos\Theta} \\ I/\Delta T, & \dfrac{ct-r-c\Delta T}{\xi-\cos\Theta} \leqslant x \leqslant \dfrac{ct-r}{\xi-\cos\Theta} \\ 0 & x>\dfrac{ct-r}{\xi-\cos\Theta} \end{cases} \tag{7}$$

as shown in Fig. 2(b) where

$$\xi=\frac{c}{c_0}, \frac{ct-r}{\xi-\cos\Theta}-\frac{ct-r-c\Delta T}{\xi-\cos\Theta}=\frac{c\Delta T}{\xi-\cos\Theta}$$

is the apparent transient width on the traveling-wave antenna. Thus (5) becomes

$$E_l\left(\frac{1}{r},\Theta,t\right)=\frac{Z_0\sin\Theta}{4\pi cr}\cdot\frac{I}{\Delta T}\int_L^U dx. \tag{8}$$

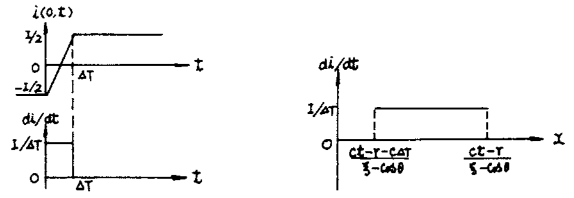
Upper and lower limits of the integration in (8) are, respectively

$$U=\begin{cases} l, & \dfrac{ct-r-c\Delta T}{\xi-\cos\Theta}\leqslant l\leqslant\dfrac{ct-r}{\xi-\cos\Theta} \\ \dfrac{ct-r}{\xi-\cos\Theta}, & 0\leqslant\dfrac{ct-r}{\xi-\cos\Theta}\leqslant l \end{cases} \tag{9}$$

$$L=\begin{cases} \dfrac{ct-r-c\Delta T}{\xi-\cos\Theta}, & 0\leqslant\dfrac{ct-r-c\Delta T}{\xi-\cos\Theta}\leqslant l \\ 0, & \dfrac{ct-r-c\Delta T}{\xi-\cos\Theta}\leqslant 0\leqslant\dfrac{ct-r}{\xi-\cos\Theta}. \end{cases} \tag{10}$$

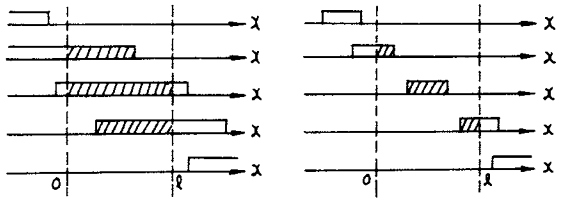
Now let us discuss the time variation of the field in the far zone when the single transient of current is traveling along the antenna.

Case I: The apparent transient width longer than the length of the antenna, i.e.,



(a) (b)

Fig. 2. (a) The waveforms of exciting current $i(0, t)$ at $x = 0$ and its derivative $di(0, t)/dt$ versus time. (b) The function $di(0, t - (x/c_0) - (r - x\cos\Theta/c))/dt$ versus x.



(a) (b)

Fig. 3. (a) Five cases of pulses traveling along the antenna when the apparent transient width is greater than the length of the antenna. (b) Five cases of pulses traveling along the antenna when the apparent transient width is smaller than the length of the antenna.

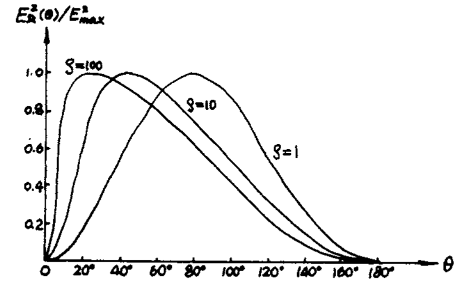


Fig. 4. Radiation patterns for a traveling-wave antenna for $\rho = 1, 10,$ and 100 and $\xi = 1$.

$$\frac{c\Delta T}{\xi-\cos\Theta}>l \quad \text{or} \quad \Theta<\Theta_c=\cos^{-1}\left(\xi-\frac{c\Delta T}{l}\right)$$

five typical positions of this transient traveling along the antenna are shown in Fig. 3(a). From (9) and (10)

$$E_l\left(\frac{1}{r},\Theta,t\right)=\begin{cases} 0 & t'<0 \\ \dfrac{Z_0I}{4\pi r}\dfrac{t'}{\Delta T}\dfrac{\sin\Theta}{\xi-\cos\Theta}, & 0\leqslant t'<\dfrac{l}{c}(\xi-\cos\Theta) \\ \dfrac{Z_0I}{4\pi r}\dfrac{l}{c\Delta T}\sin\Theta, & \dfrac{l}{c}(\xi-\cos\Theta)\leqslant t'<\Delta T \\ \dfrac{Z_0I}{4\pi r}\dfrac{1}{\Delta T}\left[\dfrac{l}{c}(\xi-\cos\Theta)+\Delta T-t'\right]\dfrac{\sin\Theta}{\xi-\cos\Theta}, & \Delta T\leqslant t'<\Delta T+\dfrac{l}{c}(\xi-\cos\Theta) \\ 0 & t'\geqslant\Delta T+\dfrac{l}{c}(\xi-\cos\Theta) \end{cases} \tag{11}$$

where

$$t'=t-\frac{r}{c}.$$

Case II: The apparent transient width shorter than the length of the antenna, i.e., $c\Delta T/(\xi - \cos\Theta) < l$ or $\Theta > \Theta_c$; five typical positions of this transient traveling along the antenna are shown in Fig. 3(b). From (9) and (10)

$$E_l\left(\frac{1}{r},\Theta,t\right) = \begin{cases} 0, & t' < 0 \\ \dfrac{Z_0 I}{4\pi r}\dfrac{t'}{\Delta T}\dfrac{\sin\Theta}{\xi - \cos\Theta}, & 0 \leqslant t' < \Delta T \\ \dfrac{Z_0 I}{4\pi r}\dfrac{\sin\Theta}{\xi - \cos\Theta}, & \Delta T \leqslant t' < \dfrac{l}{c}(\xi - \cos\Theta) \\ \dfrac{Z_0 I}{4\pi r}\dfrac{1}{\Delta T}\left[\dfrac{l}{c}(\xi - \cos\Theta) + \Delta T - t'\right]\dfrac{\sin\Theta}{\xi - \cos\Theta}, & \dfrac{l}{c}(\xi - \cos\Theta) \leqslant t' < \dfrac{l}{c}(\xi - \cos\Theta) + \Delta T \\ 0, & t' \geqslant \dfrac{l}{c}(\xi - \cos\Theta) + \Delta T. \end{cases} \tag{12}$$

From previous expressions of the field strength, we can plot the radiation waveforms in different directions. These radiation waveforms have the following characteristics (for simplicity assume $\xi = c/c_0 = 1$).

1) The radiation waveform produced by a single transient of current traveling along the antenna is a trapezoidal pulse with base width

$$W = \frac{l}{c}(1 - \cos\Theta) + \Delta T$$

where Θ is the angle between vector r and the direction of the current; it is variable from 0° to 180°. The smaller the angle Θ, the smaller the width of the space pulse.

2) The amplitude of the radiated pulse is $Z_0 I/4\pi r$ in the broadside direction ($\Theta = 90°$) for $l \geqslant c\Delta T$. As Θ decreases from 90° to Θ_c, the amplitude of the space pulse increases as predicted by $\sin\Theta/(1 - \cos\Theta)$ or $ctg\ \Theta/2$ from $Z_0 I/4\pi r$ to its maximum value

$$E_{\max} = \frac{Z_0 I}{4\pi r} ctg \frac{\Theta_c}{2} = \frac{Z_0 I}{4\pi r}\frac{l}{c\Delta T}\sin\Theta_c.$$

As Θ decreases from Θ_c to 0, the amplitude of the radiated pulse decreases again as predicted by $\sin\Theta$ from $E_{\max}$ to 0.

3) The transient times of leading and trailing edges of the space pulse are ΔT for Θ between 180° and Θ_c, and $(l/c)(1 - \cos\Theta)$ for Θ between Θ_c and 0°.

If the exciting current is not a single transient of current but a trapezoidal pulse where positive and negative transients are τ seconds apart, then the radiation waveform is a superposition of radiation waveforms of corresponding positive and negative transients. The radiation waveform of negative transient can be obtained by reversing the sign of the radiation waveform of a positive transient and shifting it to the right along the time axis by τ.

III. THE RADIATION CHARACTERISTICS OF THE SINGLE TRAVELING-WAVE ANTENNA

In the previous section, the expressions of the radiation waveform in the far zone produced by single transient of current traveling along a linear antenna were derived. If the traveling-wave antenna is excited by a trapezoidal-pulse current with a repetition rate ϕ, there are 2ϕ transients of current traveling along the antenna during 1 s. Assume there is no interference between adjacent space pulses; from (11) and (12), the mean-power pattern function for a single traveling-wave antenna is obtained

$$E_l^2\left(\frac{1}{r},\Theta\right) = \left\langle E_l^2\left(\frac{1}{r},\Theta,t\right)\right\rangle = 2\phi\int E_l^2\left(\frac{1}{r},\Theta,t\right)dt. \tag{13}$$

If $\Theta < \Theta_c$, from (11) and (13)

$$E_l^2\left(\frac{1}{r},\Theta\right) = \frac{Z_0^2 I^2 l^2}{8\pi^2 c^2 r^2}\frac{\phi}{\Delta T}\left[1 - \tfrac{1}{3}\rho(\xi - \cos\Theta)\right]\sin^2\Theta \tag{14}$$

where

$$\rho = \frac{l}{c\Delta T}.$$

If $\Theta > \Theta_c$ in (12) and (13)

$$E_l^2\left(\frac{1}{r},\Theta\right) = \frac{Z_0^2 I^2 l^2}{8\pi^2 c^2 r^2}\frac{\phi}{\Delta T}\frac{1}{\rho(\xi - \cos\Theta)} \cdot \left[1 - \frac{1}{3\rho(\xi - \cos\Theta)}\right]\sin^2\Theta. \tag{15}$$

The directional patterns of a traveling-wave antenna for $\rho = 1, 10$, and 100 are given in Fig. 4.

Let $(d/d\Theta)E_l^2(1/r,\ \Theta) = 0$; one obtains an expression of $\cos\Theta_m$ as a function of ξ and ρ. Assume $c_0 = c$, i.e., $\xi = 1$; from (14) and (15), one can find the direction of maximum radiation as follows:

$$\Theta_m = \begin{cases} \cos^{-1}\left(\frac{1}{3} - \frac{1}{\rho} + \sqrt{\frac{1}{3} + \left(\frac{1}{3} - \frac{1}{\rho}\right)^2}\right), & \rho < \frac{3}{2} \\ \cos^{-1}\left(1 - \sqrt{2/(3\rho)}\right), & \rho > \frac{3}{2}. \end{cases} \tag{16}$$

When $\xi = 1$, $\rho > 3/2$, the mean-square value of the electric field strength in the direction of maximum radiation is

$$E_l^2\left(\frac{1}{r}, \Theta_m\right) = \frac{Z_0^2 I^2 l\phi}{8\pi^2 c^2 r^2}\left(2 - \sqrt{\frac{8}{3\rho}} + \frac{1}{3\rho}\right). \tag{17}$$

The total power radiated by the traveling-wave antenna is

$$\begin{aligned} P &= \frac{1}{Z_0}\int_0^{2\pi} d\phi \int_0^{\pi} E_l^2\left(\frac{1}{r}, \Theta\right) r^2 \sin\Theta\, d\,\Theta \\ &= \frac{2\pi}{Z_0}\,\frac{Z_0^2 I^2 l^2 \phi}{8\pi^2 c^2 \Delta T}\left(2\rho^{-1} - \tfrac{5}{9}\rho^{-2} - \tfrac{1}{12}\rho^{-3}\right. \\ &\qquad \left. - \tfrac{2}{3}\rho^{-2}\ln 2\rho\right). \end{aligned} \tag{18}$$

The directivity of the traveling-wave antenna in the direction of maximum radiation is

$$D = \frac{2\left(2\rho^{-1} - 2\sqrt{\tfrac{2}{3}}\rho^{-3/2} + \tfrac{1}{3}\rho^{-2}\right)}{2\rho^{-1} - \tfrac{5}{9}\rho^{-2} - \tfrac{1}{12}\rho^{-3} - \tfrac{2}{3}\rho^{-2}\ln 2\rho},$$
$$\text{for } \xi = 1, \rho > \tfrac{3}{2}. \tag{19}$$

From the above analysis, we may conclude the following:

1) When the surrounding medium is fixed, i.e., ξ is fixed, the mean-power pattern function and the direction of maximum power radiation of the traveling-wave antenna depends only on $\rho = l/c\Delta T$, i.e., it depends on the ratio of the length of the traveling-wave antenna to the transient width of the current, but is independent of the pulse repetition rate ϕ. The greater ρ, the closer to the x-axis will be the direction of maximum radiation.

2) We know that the direction of maximum radiation of an array excited synchronously is in the broadside direction. However, the traveling-wave antenna will yield maximum radiation in the broadside direction only when its length is $c\Delta T$ or less.

3) From (12), we find that the amplitude of the space pulse is independent of the transient time ΔT if $\Theta > \Theta_c$ or $l > c\Delta T/(\xi - \cos\Theta)$. This result is different from that for the Hertzian dipole. For the Hertzian dipole, the smaller ΔT the larger the amplitude of the space pulse. The reason is that the reduction of the value of ΔT for a traveling-wave antenna will reduce the efficient radiating length, which means the integration interval will be reduced though the amplitude of di/dt will be increased in the equations for

$$E_l^2\left(\frac{1}{r}, \Theta\right).$$

4) By means of the property that the space waveforms radiated by the traveling-wave antenna vary with radiation direction, we can carry out waveform transformations to obtain a desired radiated pulse and to increase the actual radiated power. For example, assume a space pulse with a width of 1 ns is required; one may choose the exciting current with transient time $\Delta T = 0.25$ ns on purpose. If we hope to obtain a space pulse of width 1 ns in the direction of maximum radiation ($\Theta = \Theta_m$), then the stretching factor [3] is

$$\frac{\Delta t + \Delta T}{\Delta T} = 1 + \frac{l}{c\Delta T}(1 - \cos\Theta_m) = 4 \qquad \xi = 1\,. \tag{20}$$

From (16)

$$\cos\Theta_m = 1 - \sqrt{\frac{2c\Delta T}{3}} \qquad \xi = 1, \rho > \tfrac{3}{2}\,. \tag{21}$$

Solving (20) and (21), we can determine the desirable length l of the traveling-wave antenna and the direction Θ_m of maximum radiation. In this example, $l = 14\ c\Delta T$, $\Theta_m = 38°$. If an exciting current with transient time of 0.5 ns is selected, one obtains $l = c\Delta T$, $\Theta_m = 80°$. From (17) and (18), it can be found that the actual radiated power and the power density in the direction of maximum radiation for the first selection are much greater than for the latter selection.

5) The radiation efficiency of the traveling-wave antenna depends upon the ratio of the radiation resistance of the antenna to the terminal resistance. In order to increase the radiation efficiency, the terminal resistance should be reduced and the radiation resistance of the antenna should be increased. The reduction of the terminal resistance depends upon development of a pulse power generator with small internal resistance. And increasing the radiation resistance of the antenna is related to the waveform of the exciting currents.

From the derivation of the radiated power for the traveling-wave antenna, it is clear that only current transients radiate power. The trapezoidal and Rademacher currents with the same transient width the same amplitude, and the same repetition rate will radiate the same power for $l > c\Delta T$. But these currents have different mean-square values, as shown in Fig. 5. Therefore, the radiation resistance of the antenna varies with the type of exciting current.

IV. TIME-DOMAIN ANALYSIS FOR RADIATION OF THE DIPOLE ANTENNA EXCITED BY PULSE CURRENT

Consider a center-fed dipole antenna with the length l of each arm, as shown in Fig. 6. Assume a current transient with amplitude I and transient time ΔT is fed at the input terminal of the dipole. Using the results derived in Section

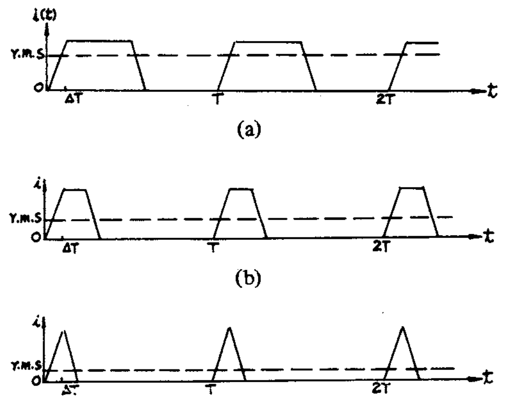


Fig. 5. Different pulse currents and their rms values: (a) Rademacher current, (b) trapezoidal pulse current, and (c) triangular pulse current.

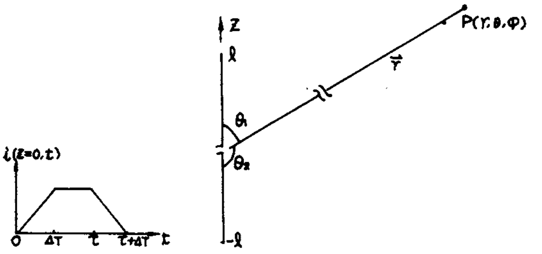


Fig. 6. Geometry of a center-fed dipole.

II, one can make time-domain analysis for the response of the dipole to a pulse.

First, we discuss the radiation due to this transient of current traveling along the upper arm of the dipole. During $0 \leq t < (l/c) + \Delta T$, the current transient at the feed point of the dipole travels along the $+z$ direction. From (11) and (12), one can plot the corresponding radiation waveform in the far zone. The amplitude of this space pulse is

$$\frac{Z_0 I}{4\pi r}\,\frac{\sin\Theta_1}{1-\cos\theta_1}, \qquad \text{for}\,\Theta_1 > \Theta_c$$

and the time duration of the pulse is

$$0 \leq t' < \frac{l}{c}\,(1-\cos\Theta_1) + \Delta T.$$

When the current transient arrives at the end of the upper arm ($z = l$), there will be a reflection of the current and the polarity of the transient will be reversed.

Due to the end effect of the antenna, the current reflection coefficient may be taken as $\Gamma_e = -0.9$. Thus during the time period $(l/c) \leq t < (2l/c) + \Delta T$, there is a current transient with opposite polarity traveling along the $-z$ direction from the open end of the upper arm as well. The polarity of the corresponding space pulse will be reversed. Because Θ becomes $180^\circ - \Theta_1 = \Theta_2$, from (11) and (12) we know the amplitude of the space pulse becomes

$$\frac{Z_0\Gamma_e I}{4\pi r}\,\frac{\sin\Theta_2}{1-\cos\Theta_2} = \frac{Z_0\Gamma_e I}{4\pi r}\,\frac{\sin\Theta_1}{1+\cos\Theta_1}, \qquad \text{for}\,\Theta_2 > \Theta_c$$

and the time duration of the pulse is

$$\frac{l}{c}\,(1-\cos\Theta_1) \leq t' < \frac{2l}{c} + \Delta T.$$

When the current transient arrives at the feed point, a second reflection occurs. Assume the antenna impendance is Z_A,[1] feeder impedance is Z_s, then current reflection coefficient $\Gamma_f = (Z_A - Z_s)/(Z_A + Z_s)$. Therefore, since $t = 2l/c$, there is a current transient with the amplitude $I\Gamma_e\Gamma_f$ traveling along $+z$ direction. (If Γ_f is positive, the polarity of the reflection current will not be reversed.) By analogy, the set of these corresponding space pulses forms the time-domain waveform of the field radiated by a single current transient traveling back and forth on the upper arm of the dipole.

If the exciting current is a trapezoidal pulse where the leading- and trailing-edge width is ΔT, respectively, and base width is $\tau + \Delta T$, for the radiation field in the far zone it is equivalent to a positive transient arising at $t = 0$ plus a negative transient arising at $t = \tau$. Therefore, the radiation waveform due to the trapezoidal current equals the superposition of the radiation waveforms of corresponding positive and negative transients.

For the radiation produced by the pulse current transient on the lower arm of the dipole, there is a similar analysis, except that the incident positive transient of current now travels along the $-z$ direction; then $\Theta = \Theta_2$. After first reflection at the open end of the dipole, it travels along the $+z$ direction and $0 = \Theta_1$.

By superposition of the radiation waveforms produced by the current transient in the upper arm and the lower arm of the dipole, the time-domain response of the dipole antenna to the pulse current is obtained.

Clearly, the pulse response of the monopole over an infinite conductive plane can be analyzed by the same means, because the radiation field produced by it is the same as that produced by a corresponding dipole.

Let us consider an example. Schmitt *et al.* [4] and Palciauskas *et al.* [13] performed frequency-domain analyses and experimental demonstrations for pulse response of a practical dipole. In order to compare the results easily, the time-domain analyses in this paper will be performed under the same conditions, i.e., the dipole discussed is a center-fed cylindrical thin antenna with the arm-length-to-radius ratio $l/a = 904$, the exciting signal is a trapezoidal current pulse with a base width of about 0.55 l/c, the transient time $\Delta T = 0.15\ l/c$ and the time interval between positive and negative transients $\tau = 0.40\ l/c$. Assume the current reflection coefficient at the open end of the dipole $\Gamma_e = -0.9$, the antenna impendance $Z_A = 300\ \Omega$, the feeder impedance $Z_f = 50\ \Omega$, and the current reflection coefficient at the feed point $\Gamma_f = (300 - 50)/(300 + 50) = 0.72$, as shown in [13].

According to the results derived in Section II and the time-domain method advanced in Section IV, the radiation waveform produced by this practical dipole can be plotted. Fig. 7 shows the waveform of the space pulse in the $\Theta = 60^\circ$

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[1] The definition of the impedance of the antenna driven by pulse current is taken from [15].

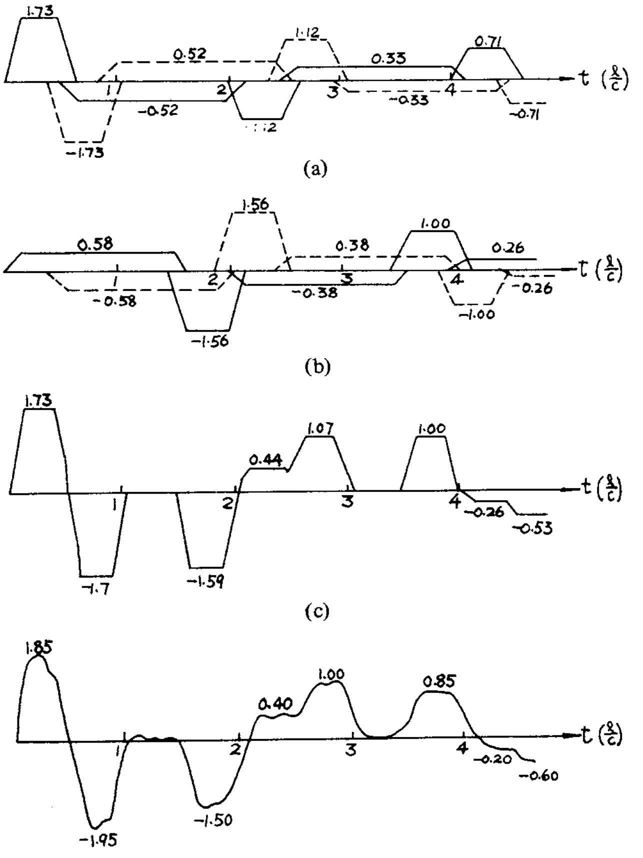

Fig. 7. The radiation waveform of a dipole excited by pulse current in the direction $\Theta = 60^\circ$. (a) The radiation of the upper arm of the dipole. (b) The radiation of the lower arm of the dipole. (c) The radiation waveform of the dipole obtained from time-domain analysis. (d) The radiation waveform of the dipole obtained from frequency-domain analysis in [13].

direction. In this figure, (a) shows the radiation waveforms in the far zone produced by the upper arm of the dipole, where solid lines represent the radiation waveform due to the leading edge of the exciting pulse and the dashed lines represent the radiation waveform due to the trailing edge of the exciting pulse; the total radiation waveform of the upper arm of the dipole can be obtained by superposing the solid and the dashed lines. Fig. 7(b) shows the radiation waveform produced by the lower arm of the dipole. In Fig. 7(a) and (b), the marked amplitude values are the calculated values of the factors except $Z_0 I/4\pi r$ in (12). Fig. 7(c) shows the superposition of the radiation waveforms produced by upper and lower arms of the dipole, i.e., the time-domain waveform of the radiation produced by the dipole. The amplitude values marked in Fig. 7(c) are obtained by adding together the corresponding values marked in (a) and (b), and multiplying by a factor 0.75, so that the first space pulse has the same amplitude as the corresponding pulse in [13], thereby affording ready comparison. Fig. 7(d) represents this radiation waveform in the $\Theta = 60^\circ$ direction obtained from frequency-domain analysis in [13]. When the waveform of the time-domain analysis in Fig. 7(c) is compared with the corresponding waveform of the frequency-domain analysis in Fig. 7(d), it is clear that there is good agreement between these two results.

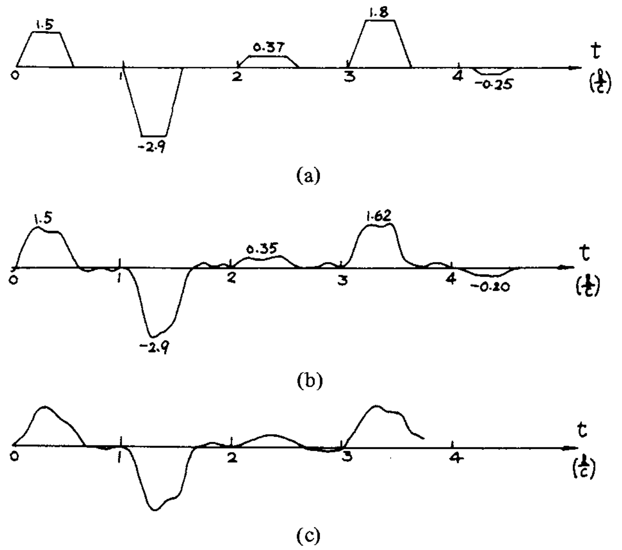

Fig. 8. The radiation waveform of a dipole excited by pulse current in the direction $\Theta = 90^\circ$ (broadside direction). (a) The radiation waveform of a dipole obtained from time-domain analysis. (b) The radiation waveform of a dipole obtained from frequency-domain analysis in [13]. (c) The radiation waveform of a dipole measured in [4].

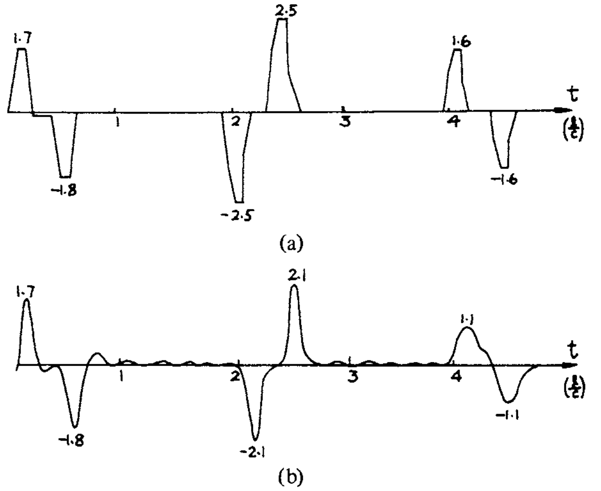

Fig. 9. The radiation waveform of a dipole excited by pulse current in the direction $\theta = 20^\circ$. (a) The radiation waveform of a dipole obtained from time-domain analysis. (b) The radiation waveform of a dipole obtained from frequency-domain analysis in [13].

In Fig. 8(a) is shown the results of time-domain analysis for the radiation waveform of the dipole in the $\Theta = 90^\circ$ direction (broadside direction); (b) represents the corresponding results of frequency-domain analysis quoted from [13]; and (c) represents the corresponding measured data quoted from [4]. Clearly, there is good agreement among these results obtained by three different ways.

Fig. 9 represents the radiation waveforms of the dipole in the $\Theta = 20^\circ$ direction. Similarly, there is good agreement between the results obtained from time-domain analysis and those from frequency-domain analysis as well.

In other radiation directions, similar satisfactory agreements can also be obtained. These are not listed in detail here.

In 1972, Handelsman [14] attempted to make a time-domain analysis for the radiation of a dipole excited by pulse current. Unfortunately, there are some errors in his report:

1) The radiation waveform produced by a current transient with a finite slope is a trapezoidal pulse with finite slope (see (11), (12), and the results in Section II). But in [14], Handelsman has a rectangular pulse. Thus some inappropriate results occurred in his paper, such as the radiation waveform of the dipole being a series of rectangular pulses [14, pp. 66-68, fig. 32].

2) The expressions of radiation waveforms for $\Theta < \Theta_c$ are different from that for $\Theta > \Theta_c$ (see (11) and (12) of Section II). But in [14], this important fact was neglected, leading to a strange result that the amplitude of the radiation waveform will become infinite as $\Theta \rightarrow 0$ (i.e., approaching the endfire direction). Because of the previous reasonings, the radiation waveforms of the dipole for $\Theta < \Theta_c$ given in [14] (e.g., for $\Theta = 20°$, see [14, fig. 32(c)]) are quite different from that given by frequency-domain analysis in [13].

These errors in [14] resulted principally from inappropriate treatment of [14, eq. (99)] in that report. The integral on the left of [14, eq. (99)] should be calculated in each interval of time for different ranges of the value of Θ (see Section II). From (11) and (12), it is clear that there are no such strange phenomena as occurred in [14]. In fact, the amplitude of the radiation waveform produced by the dipole will become zero as $\Theta \rightarrow 0$.

V. CONCLUSIONS

From the previous discussion, we may conclude the following:

The fields radiated by traveling-wave antennas, dipoles, and monopoles over an infinite conductive plane excited by pulse currents can be obtained from time-domain analysis. The basis of this analysis is the expression of the radiation waveforms in the far zone produced by a single transient of current traveling along a thin straight wire derived in Section II.

When the surrounding medium is fixed, the mean-power pattern function and the direction of maximum power radiation of the traveling-wave antenna depends only on $\rho = l/c\Delta T$, i.e., it depends only upon the ratio of the length of the traveling-wave antenna to the transient width of the current, but is independent of the pulse repetition rate ϕ. The greater ρ, the closer to the x-axis will be the direction of maximum radiation.

The radiation waveform of the traveling-wave antenna excited by a linear transient of current is a "clear" trapezoidal pulse. But the radiation waveform of the dipole excited by linear transients of current is a series of pulses. In video-probing techniques, this result greatly deteriorates the resolution of targets.

There are good agreements between results of time-domain analysis and frequency-domain analysis for radiation waveforms in different directions produced by the dipole. These verify the results for the radiation waveform of a single transient of current and the radiation characteristics of the traveling-wave antenna given in this paper.

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